Nuclear structure functions

Ian Cloët
(University of Washington)

Collaborators

Wolfgang Bentz
(Tokai University)

Anthony Thomas
(Adelaide University)

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Aspects of DIS on nuclear targets $\rightarrow$ nuclear structure functions
- Highlight opportunities provided by nuclear systems to study QCD
- Gain insight into nuclear structure from a QCD viewpoint

Present complementary approach to traditional nuclear physics
- formulated as a covariant quark theory
- grounded in good description of mesons and baryons
- at finite density self-consistent mean-field approach
- bound nucleons differ from free nucleons

Possible answers to many long standing questions: we address
- EMC effect & NuTeV anomaly

Highlight the unique opportunities provided by PV DIS on nuclei
EMC Effect


**Immediate parton model interpretation:**
- Valence quarks in nucleus carry less momentum than in nucleon
- Nuclear effects seem to influence the quarks in the bound nucleons
- What is the mechanism? After 25 years no consensus
- EMC $\Rightarrow$ medium modification
Medium Modification

- 50 years of traditional nuclear physics tells us that the nucleus is composed of nucleon-like objects
- However if a nucleon property is not protected by a symmetry its value may change in medium – for example:
  - mass, magnetic moment, size
  - quark distributions, form factors, GPDs, etc
- There must be medium modification:
  - nucleon propagator is changed in medium
  - off-shell effects \((p^2 \neq M^2)\)
  - Lorentz covariance implies bound nucleon has 12 EM form factors

\[
\langle J^\mu \rangle = \sum_{\alpha, \beta=+,-} \Lambda^\alpha(p') \left[ \gamma^\mu f_1^{\alpha\beta} + \frac{1}{2M} i \sigma^{\mu\nu} q_\nu f_2^{\alpha\beta} + q^\mu f_3^{\alpha\beta} \right] \Lambda^\beta(p)
\]

- Need to understand these effects as first step toward QCD based understanding of nuclei
Medium Modification

- 50 years of traditional nuclear physics tells us that the nucleus is composed of nucleon-like objects.

- However if a nucleon property is not protected by a symmetry its value may change in medium – for example:
  - mass, magnetic moment, size
  - quark distributions, form factors, GPDs, etc

- There must be medium modification:
  - nucleon propagator is changed in medium
  - off-shell effects \((p^2 \neq M^2)\)
  - Becomes 2 form factors for on-shell nucleon

\[
\langle J^\mu \rangle = \bar{u}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{1}{2M} i \sigma^{\mu\nu} q_\nu F_2(Q^2) \right] u(p)
\]

- Need to understand these effects as first step toward QCD based understanding of nuclei.
Why nuclear targets?
- only targets with $J \geq 1$ are nuclei
- study QCD and nucleon structure at finite density

Hadronic Tensor: in Bjorken limit & Callen-Gross ($F_2 = 2x F_1$)
- For $J = \frac{1}{2}$ target

\[
W_{\mu\nu} = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F_2(x, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_1(x, Q^2)
\]

- For arbitrary $J$: $-J \leq H \leq J$ \([2J + 1 \text{ structure functions}]

\[
W^H_{\mu\nu} = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F_{2A}^H(x_A, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_{1A}^H(x_A, Q^2)
\]

Parton model expressions \([2J + 1 \text{ quark distributions}]

\[
F_{2A}^H(x_A) = \sum_q e_q^2 x_A \left[ q_A^H(x_A) + \bar{q}_A^H(x_A) \right]; \quad \text{parity} \quad \Rightarrow \quad F_{2A}^H = F_{2A}^{-H}
\]
**DIS on Nuclear Targets**

- **Hadronic Tensor:** in Bjorken limit & Callen-Gross \((F_2 = 2x F_1)\)
  - For \(J = \frac{1}{2}\) target
    \[
    W_{\mu\nu} = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_{\mu}p_{\nu}}{p \cdot q} \right) F_2(x, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_1(x, Q^2)
    \]
  - For arbitrary \(J\): \(-J \leq H \leq J\)  \([2J + 1\) structure functions]\]
    \[
    W_{\mu\nu}^H = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_{\mu}p_{\nu}}{p \cdot q} \right) F_{2A}^H(x_A, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_{1A}^H(x_A, Q^2)
    \]

- **Parton model expressions** \([2J + 1\) quark distributions]
  \[
  F_{2A}^H(x_A) = \sum_q e_q^2 x_A \left[ q_A^H(x_A) + \overline{q}_A^H(x_A) \right] ; \text{ parity } \implies F_{2A}^H = F_{2A}^{-H}
  \]
  \[
  F_{2A}(x) = \frac{1}{2J + 1} \sum_{H=-J}^J F_{2A}^H(x)
  \]
Finite nuclei quark distributions

- **Definition of finite nuclei quark distributions**

\[ q^H_A (x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^- / A} \langle A, P, H | \bar{\psi}_q(0) \gamma^+ \psi_q(\xi^-) | A, P, H \rangle \]

- **Approximate using a modified convolution formalism**

\[ q^H_A (x_A) = \sum_{\alpha,\kappa,m} \int dy_A \int dx \, \delta(x_A - y_A x) f^{(H)}_{\alpha,\kappa,m}(y_A) \, q_{\alpha,\kappa}(x) \]
Finite nuclei quark distributions

- **Definition of finite nuclei quark distributions**

\[
q^H_A (x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^-/A} \langle A, P, H | \bar{\psi}_q(0) \gamma^+ \psi_q(\xi^-) | A, P, H \rangle
\]

- **Approximate using a modified convolution formalism**

\[
q^H_A (x_A) = \sum_{\alpha, \kappa, m} \int dy_A \int dx \, \delta(x_A - y_A x) f^{(H)}_{\alpha, \kappa, m}(y_A) \, q_{\alpha, \kappa}(x)
\]

- **Convolution formalism diagrammatically:**

![Convolution formalism diagram](image-url)
Assume all spin is carried by the valence nucleons

- If \( A \gtrsim 8 \) and for example if: \( J = \frac{3}{2} \Rightarrow F_{2A}^{3/2} \simeq F_{2A}^{1/2} \)

This is a model independent result within the convolution formalism

- Introduce multipole quark distributions

\[
q^{(K)}(x) = \sum_{H} (-1)^{J-H} \sqrt{2K+1} \left( \frac{J}{H} \frac{J}{H} \frac{K}{0} \right) q^{H}(x), \quad K = 0, 2, \ldots, 2J
\]

- Example: \( J = \frac{3}{2} \longrightarrow q^{(0)} = q^{\frac{3}{2}} + q^{\frac{1}{2}} \quad q^{(2)} = q^{\frac{3}{2}} - q^{\frac{1}{2}} \)

- Higher multipoles encapsulate difference between helicity distributions
Some multipole quark distributions result

- Large $K > 1$ multipole PDFs would be very surprising
- large off-shell effects &/or non-nucleon components, etc
New Sum Rules

- Sum rules for multipole quark distributions

\[ \int dx \, x^{n-1} \, q^{(K)}(x) = 0, \quad K, n \text{ even}, \quad 2 \leq n < K, \]
\[ \int dx \, x^{n-1} \, \Delta q^{(K)}(x) = 0, \quad K, n \text{ odd}, \quad 1 \leq n < K. \]

- Examples:

\[ J = \frac{3}{2} \implies \langle \Delta q^{(3)}(x) \rangle = 0 \]
\[ J = 2 \implies \langle \Delta q^{(3)}(x) \rangle = \langle q^{(4)}(x) \rangle = 0 \]
\[ J = \frac{5}{2} \implies \langle \Delta q^{(3)}(x) \rangle = \langle q^{(4)}(x) \rangle = \langle \Delta q^{(5)}(x) \rangle = \langle x^2 \Delta q^{(5)}(x) \rangle = 0 \]

- Sum rules place tight constraints on multipole PDFs

Nambu–Jona-Lasinio Model

- Interpreted as low energy chiral effective theory of QCD
- Can be motivated by infrared enhancement of gluon propagator, e.g. DSEs and Lattice QCD
- Investigate the role of quark degrees of freedom.
- NJL has same symmetries as QCD
- Lagrangian: \( \mathcal{L}_{NJL} = \bar{\psi} \left( i \partial - m \right) \psi + G (\bar{\psi} \Gamma \psi)^2 \)
Nucleon in the NJL model

- Nucleon approximated as quark-diquark bound state
- Use relativistic Faddeev approach:

\[
\begin{align*}
P - k &= P - k, \\
\end{align*}
\]

- Nucleon quark distributions

\[
q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s|\bar{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s \rangle_c, \quad \Delta q(x) = \langle \gamma^+\gamma_5 \rangle
\]

- Associated with a Feynman diagram calculation

\[
[q(x), \Delta q(x), \Delta_T q(x)] \to \mathbf{X} = \delta \left( x - \frac{k^+}{p^+} \right) \left[ \gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5 \right]
\]
Results: proton quark distributions

Empirical distributions:


NJL model gives good description of free nucleon quark distributions

Approach is covariant, satisfies all sum rules & positivity constraints

DGLAP equations [Dokshitzer (1977), Gribov-Lipatov (1972), Altarelli-Parisi (1977)]

\[ \frac{\partial}{\partial \ln Q^2} q_v(x, Q^2) = \alpha_s(Q^2) P(z) \otimes q_v(y, Q^2) \]
Asymmetric Nuclear Matter

- **Fundamental physics**: mean fields couple to the quarks in nucleons

- **Finite density mean-field Lagrangian**: $\sigma$, $\omega$, $\rho$ fields

\[ L = \overline{\psi} \left( i \not{\partial} - M^* - \not{V} \right) \psi + L'_I \]

- $\sigma$: isoscalar-scalar – attractive
- $\omega$: isoscalar-vector – repulsive
- $\rho$: isovector-vector – attractive/repulsive

- **Finite density quark propagator**

\[ S(k)^{-1} = \mathbf{k} - M - i\varepsilon \quad \rightarrow \quad S_q(k)^{-1} = \mathbf{k} - M^* - V_q - i\varepsilon \]
Effective Potential

- Hadronization → Effective potential

\[ \mathcal{E} = \mathcal{E}_V - \frac{\omega_0^2}{4G_\omega} - \frac{\rho_0^2}{4G_\rho} + \mathcal{E}_p + \mathcal{E}_n \]

- \( \mathcal{E}_V \): vacuum energy
- \( \mathcal{E}_{p(n)} \): energy of nucleons moving in \( \sigma, \omega, \rho \) fields

- Effective potential provides

\[ \omega_0 = 6 G_\omega (\rho_p + \rho_n), \quad \rho_0 = 2 G_\rho (\rho_p - \rho_n), \quad \frac{\partial \mathcal{E}}{\partial M^*} = 0 \]

- \( G_\omega \leftrightarrow Z = N \) saturation & \( G_\rho \leftrightarrow \) symmetry energy

- Quark vector fields:

\[ V_{u(d)} = \omega_0 \pm \rho_0 \]

- Recall: quark propagator:

\[ S_q(k) = \left[ k^2 - M^* - V_q \right]^{-1} \]
**Isovector EMC effect**

**EMC ratio:**
\[ R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z F_{2p} + N F_{2n}} \approx \frac{4 u_A(x) + d_A(x)}{4 u_0(x) + d_0(x)} \]

- Density is fixed only \( Z/N \) ratio is changing
  - non-trivial isospin dependence
- Proton excess: \( u \)-quarks feel more repulsion than \( d \)-quarks
- Neutron excess: \( d \)-quarks feel more repulsion than \( u \)-quarks
- Isovector interaction \( \rightarrow \) isovector EMC Effect
Weak mixing angle and the NuTeV anomaly

- **NuTeV**: \( \sin^2 \theta_W = 0.2277 \pm 0.0013 \text{(stat)} \pm 0.0009 \text{(syst)} \)
- **World average**: \( \sin^2 \theta_W = 0.2227 \pm 0.0004 \) : \( 3 \sigma \) \( \Rightarrow \) “NuTeV anomaly”
- Huge amount of experimental & theoretical interest [over 400 citations]
- No universally accepted complete explanation
**Paschos-Wolfenstein ratio**

- Paschos-Wolfenstein ratio motivated the NuTeV study:

\[
R_{PW} = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\bar{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\bar{\nu} A}}, \quad NC \Rightarrow Z^0, \quad CC \Rightarrow W^\pm
\]

- Expressing \( R_{PW} \) in terms of quark distributions:

\[
R_{PW} = \left( \frac{1}{6} - \frac{4}{9} \sin^2 \theta_W \right) \langle x u_A \rangle + \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_W \right) \langle x d_A + x s_A \rangle
\]

\[
\langle x d_A + x s_A \rangle - \frac{1}{3} \langle x u_A \rangle
\]

- For an isoscalar target \( u_A \simeq d_A \) and if \( s_A \ll u_A + d_A \)

\[
R_{PW} = \frac{1}{2} - \sin^2 \theta_W + \Delta R_{PW}
\]

- NuTeV measured \( R_{PW} \) on an Fe target \( (Z/N \simeq 26/30) \)

- Correct for neutron excess \( \Leftrightarrow \) isoscalarity corrections
Isovector EMC correction to NuTeV

- General form of isoscalarity corrections

\[ R_{PW} = \left( \frac{1}{2} - \sin^2 \theta_W \right) + \left( 1 - \frac{7}{3} \sin^2 \theta_W \right) \frac{\langle x u_A - x d_A \rangle}{\langle x u_A + x d_A \rangle} \]

- NuTeV assumed nucleons in Fe are like free nucleons
  - Ignored some medium effects: Fermi motion & \( \rho^0 \)-field

- Use our medium modified “Fe” quark distributions

\[ \Delta R_{PW} = \Delta R_{PW}^{\text{naive}} + \Delta R_{PW}^{\text{Fermi}} + \Delta R_{PW}^{\rho^0} = -(0.0107 + 0.0004 + 0.0028). \]

- Recall NuTeV requires \( \Delta R_{PW} = -0.005 \)

\[ R_{PW}^{\text{SM}} \equiv 0.2773 \pm \ldots \ (= \frac{1}{2} - \sin^2 \theta_W) \]

\[ R_{PW}^{\text{NuTeV}} = 0.2723 \pm \ldots \]

- Isoscalarity \( \rho^0 \) correction can explain up to 65% of anomaly
**NuTeV anomaly cont’d**

- Also correction from $m_u \neq m_d$ - Charge Symmetry Violation
  - CSV + $\rho_0$ $\implies$ no NuTeV anomaly
  - No evidence for physics beyond the Standard Model

- Instead “NuTeV anomaly” is evidence for medium modification
  - Equally interesting
  - EMC effect has over 850 citations

- Model dependence?
  - sign of correction is fixed by nature of vector fields
    \[
    q(x) = \frac{p^+}{p^+} q_0 \left( \frac{p^+}{p^+} - \frac{V_q^+}{p^+} \right), \quad N > Z \implies V_d > V_u
    \]
  - $\rho^0$-field shifts momentum from $u$- to $d$-quarks
  - size of correction is constrained by Nucl. Matt. symmetry energy

- $\rho_0$ vector field reduces NuTeV anomaly – Model Independent!!
Total NuTeV correction

- Includes NuTeV functionals
- Small increase in systematic error
- NuTeV anomaly interpreted as evidence for medium modification
- Equally profound as evidence for physics beyond Standard Model
**Consistent with other observables?**

- We claim isovector EMC effect explains $\sim 1.5\sigma$ of NuTeV result
  - is this mechanism observed elsewhere?

- Yes!! Parity violating DIS: $\gamma Z^0$ interference

\[
A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \propto \left[ a_2(x) + \frac{1 - (1 - y)^2}{1 + (1 - y)^2} a_3(x) \right]
\]

\[
a_2(x) = -2g^e_A \frac{F_2^{\gamma Z}}{F_2^\gamma} = \frac{6u^+ + 3d^+}{4u^+ + d^+} - 4\sin^2 \theta_W
\]

\[
a_3(x) = -2g^e_V \frac{F_3^{\gamma Z}}{F_2^\gamma} = 3 \left( 1 - 4\sin^2 \theta_W \right) \frac{2u^- + d^-}{4u^+ + d^+}
\]

- Parton model expressions

\[
F_2^{\gamma Z} = 2 \sum e_q g^q_V \ x (q + \bar{q}) , \quad g^q_V = \pm \frac{1}{2} - 2e_q \sin^2 \theta_W
\]

\[
F_3^{\gamma Z} = 2 \sum e_q g^q_A \ (q - \bar{q}) , \quad g^q_A = \pm \frac{1}{2}
\]
**Parity Violating DIS: Carbon**

- Ignoring quark mass differences, $s$-quarks and EW corrections
  - For a $N = Z$ target:

  $$a_2(x) = \frac{6u_A^+ + 3d_A^+}{4u_A^+ + d_A^+} - 4\sin^2\theta_W \rightarrow \frac{9}{5} - 4\sin^2\theta_W$$

  $$a_3(x) = 3\left(1 - 4\sin^2\theta_W\right)\frac{2u^- + d^-}{4u_A^+ + d_A^+} \rightarrow \frac{9}{5}\left(1 - 4\sin^2\theta_W\right)\frac{u_A^- + d_A^-}{u_A^+ + d_A^+}$$

- Measurement of $a_2(x)$ at each $x$ $\implies$ a NuTeV experiment!
**Parity Violating DIS: Carbon**

- Ignoring quark mass differences, $s$-quarks and EW corrections
  - For a $N = Z$ target:

  $$a_2(x) = \frac{6u_A^+ + 3d_A^+}{4u_A^+ + d_A^+} - 4\sin^2\theta_W \rightarrow \frac{9}{5} - 4\sin^2\theta_W$$

  $$a_3(x) \rightarrow \frac{9}{5} (1 - 4\sin^2\theta_W) \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} = \frac{9}{5} (1 - 4\sin^2\theta_W) \left[1 + 2 \frac{\bar{u}_A + \bar{d}_A}{u_A^- + d_A^-}\right]^{-1}$$

- Measurement of $a_2(x)$ at each $x \implies$ a NuTeV experiment!
**Parity Violating DIS: Iron**

- For a $N \approx Z$ target:

$$a_2(x) = \frac{9}{5} - 4 \sin^2 \theta_W - \frac{12}{25} \frac{u_A^+(x) - d_A^+(x)}{u_A^+(x) + d_A^+(x)}$$

$$a_3(x) = \frac{9}{5} (1 - 4 \sin^2 \theta_W) \left\{ \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} - \frac{1}{3} \left[ \frac{12}{5} \frac{u_A^-+d_A^-}{u_A^++d_A^+} \frac{u_A^+-d_A^+}{u_A^++d_A^+} - \frac{u_A^-d_A^-}{u_A^+d_A^+} \right] \right\}$$

- “Naive” result has no medium corrections

- Sizeable medium effects in $a_2(x)$
For a $N \simeq Z$ target:

$$a_2(x) = \frac{9}{5} - 4 \sin^2 \theta_W - \frac{12}{25} \frac{u_A^+(x) - d_A^+(x)}{u_A^+(x) + d_A^+(x)}$$

$$a_3(x) = \frac{9}{5} \left(1 - 4 \sin^2 \theta_W \right) \left\{ \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} - \frac{1}{3} \left[ \frac{12}{5} \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} \frac{u_A^+ - d_A^+}{u_A^+ + d_A^+} - \frac{u_A^- - d_A^-}{u_A^+ + d_A^+} \right] \right\}$$

After naive isoscalarity corrections medium effects still very large

Large $x$ dependence of $a_2(x) \rightarrow$ evidence for medium modification
Finite nuclei EMC effects

- **EMC ratio**

\[
R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z F_{2p} + N F_{2n}}
\]

- **Polarized EMC ratio**

\[
R_{s}^{H} = \frac{g_{1A}^{H}}{g_{1A}^{\text{H,naive}}} = \frac{g_{1A}^{H}}{P_{p}^{H} g_{1p} + P_{n}^{H} g_{1n}}
\]

- **Spin-dependent cross-section is suppressed by \(1/A\)**
  - Must choose nuclei with \(A \lesssim 27\)
  - Protons should carry most of the spin e.g. \(\rightarrow ^{7}\text{Li}, ^{11}\text{B}, \ldots\)

- **Ideal nucleus is probably \(^{7}\text{Li}\)**
  - From Quantum Monte–Carlo: \(P_{p}^{J} = 0.86\) & \(P_{n}^{J} = 0.04\)

- **Ratios equal 1 in non-relativistic and no-medium modification limit**
EMC ratio $^7\text{Li}$, $^{11}\text{B}$, $^{15}\text{N}$ and $^{27}\text{Al}$

![Graphs showing EMC ratios for $^7\text{Li}$, $^{11}\text{B}$, $^{15}\text{N}$, and $^{27}\text{Al}$ with $Q^2 = 5\text{ GeV}^2$.](image-url)
Recall EMC ratio:

\[ R^i = \frac{F^i_{2A}}{F^{i,\text{naive}}_{2A}} = \frac{F^i_{2A}}{Z F^i_{2p} + N F^i_{2n}} \quad i \in \gamma, \gamma Z, \ldots \]

\[ R^\gamma \sim \frac{4 u_A(x) + d_A(x)}{4 u_0(x) + d_0(x)}, \quad R^{\gamma Z} \sim \frac{1.16 u_A(x) + d_A(x)}{1.16 u_0(x) + d_0(x)} \]
\(F_2^\gamma\) and \(F_2^{\gamma Z}\) EMC ratios – “Iron” & “Lead”

\[
R^\gamma \sim \frac{4u_A(x) + d_A(x)}{4u_0(x) + d_0(x)} \quad \& \quad R^{\gamma Z} \sim \frac{1.16 u_A(x) + d_A(x)}{1.16 u_0(x) + d_0(x)}
\]

- \(u_A\) dominates \(R^\gamma\) where \(R^{\gamma Z}\) almost isoscalar ratio

- \(N > Z\): \(d\)-quarks feel more repulsion than \(u\)-quarks: \(V_d > V_u\)

\[
q(x) = \frac{p^+}{p^+ - V^+} q_0 \left( \frac{p^+}{p^+ - V^+} x - \frac{V_q^+}{p^+ - V^+} \right)
\]

- \(\rho_0\)-field \(\Rightarrow \) \(R^{\gamma Z} > R^\gamma\) – Model Independent
Flavour Dependence of EMC effect

- Flavour dependence determined by measuring $F_{2A}^{\gamma}$ and $F_{2A}^{\gamma Z}$
- Defined above by

$$R_A^q = \frac{F_{2A}^{q, \text{naive}}}{F_{2A}^{q}} \sim \frac{q_A}{q_0}$$

- If observed $\Rightarrow$ very strong evidence for medium modification
Is there medium modification

$^{27}\text{Al}$

EMC Ratios

$Q^2 = 5 \text{ GeV}^2$

Experiment: $^{27}\text{Al}$

Unpolarized EMC effect

Polarized EMC effect
Is there medium modification

- Medium modification of nucleon has been switched off
- Relativistic effects remain
- Large splitting would be strong evidence for medium modification
## Nuclear Spin Sum

<table>
<thead>
<tr>
<th>Proton spin states</th>
<th>$\Delta u$</th>
<th>$\Delta d$</th>
<th>$\Sigma$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0.97</td>
<td>-0.30</td>
<td>0.67</td>
<td>1.267</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>0.91</td>
<td>-0.29</td>
<td>0.62</td>
<td>1.19</td>
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<tr>
<td>$^{11}\text{B}$</td>
<td>0.88</td>
<td>-0.28</td>
<td>0.60</td>
<td>1.16</td>
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<tr>
<td>$^{15}\text{N}$</td>
<td>0.87</td>
<td>-0.28</td>
<td>0.59</td>
<td>1.15</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
<td>0.87</td>
<td>-0.28</td>
<td>0.59</td>
<td>1.15</td>
</tr>
<tr>
<td>Nuclear Matter</td>
<td>0.79</td>
<td>-0.26</td>
<td>0.53</td>
<td>1.05</td>
</tr>
</tbody>
</table>

- Angular momentum of nucleon: \[ J = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + J_g \]
  - in medium $M^* < M$ and therefore quarks are more relativistic
  - lower components of quark wavefunctions are enhanced
  - quark lower components usually have larger angular momentum
  - $\Delta q(x)$ very sensitive to lower components

- Conclusion: quark spin $\rightarrow$ orbital angular momentum in-medium
Conclusion

- Effective quark theories can be used to incorporate quarks into a traditional description of nuclei
  - complementary approach to traditional nuclear physics

- Major outstanding discrepancy with Standard Model predictions for $Z^0$ is NuTeV anomaly
  - may be resolved by CSV and isovector EMC effect corrections

- EMC effect and NuTeV anomaly are interpreted as evidence for medium modification of the bound nucleon wavefunction

- This result can be tested using PV DIS measurements
  - predict large medium modification in PV DIS
  - predict flavour dependence of EMC effect can be large

- Tried to demonstrate that PV DIS is a very powerful tool with which to study the parton structure of nuclei – at JLab or an EIC
Free Parameters:
\[ \Lambda_{IR}, \Lambda_{UV}, M_0, G_\pi, G_s, G_a, G_\omega \text{ and } G_\rho \]

Constraints:
- \( f_\pi = 93 \text{ MeV}, \ m_\pi = 140 \text{ MeV} \) & \( M_N = 940 \text{ MeV} \)
- \( \int_0^1 dx \ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267 \)
- \( (\rho, E_B/A) = (0.16 \text{ fm}^{-3}, -15.7 \text{ MeV}) \)
- \( a_4 = 32 \text{ MeV} \)
- \( \Lambda_{IR} = 240 \text{ MeV} \)

We obtain [MeV]:
- \( \Lambda_{UV} = 644 \)
- \( M_0 = 400, \ M_s = 690, \ M_a = 990, \ldots \)

Can now study a very large array of observables:
- e.g. meson and baryon: quark distributions, form factors, GPDs, finite temp. and density, neutron stars


**Regularization**

- **Proper-time regularization**

\[
\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau \tau^{n-1} e^{-\tau X} \quad \longrightarrow \quad \frac{1}{(n-1)!} \int_{1/(\Lambda_{IR})^2}^{1/(\Lambda_{UV})^2} d\tau \tau^{n-1} e^{-\tau X}
\]

- \(\Lambda_{IR}\) eliminates unphysical thresholds for the nucleon to decay into quarks: → simulates confinement
  

- **E.g.: Quark wave function renormalization**
  
  ✦ \(Z(k^2) = e^{-\Lambda_{UV}(k^2-M^2)} - e^{-\Lambda_{IR}(k^2-M^2)}\)
  
  \(\rightarrow\) \(Z(k^2 = M^2) = 0 \quad \Longrightarrow \quad \text{no free quarks}\)

- **Needed for:** nuclear matter saturation, \(\Delta\) baryon, etc
  
Gap Equation & Mass Generation

\[
\frac{-1}{\pi} = \frac{-1}{\rho} + \frac{1}{\rho - m + i\varepsilon} + \frac{1}{\rho - M + i\varepsilon}
\]

- **Quark Propagator:**
  \[
  \frac{1}{p - m + i\varepsilon} \rightarrow \frac{1}{p - M + i\varepsilon}
  \]

- **Mass is generated via interaction with vacuum**

- **Dynamically generated quark masses**
  \[\langle \bar{\psi}\psi \rangle = 0 \quad \leftrightarrow \quad \text{Dynamical Quark Masses} \]

- **Proper-time regularization:** \(\Lambda_{IR}\) and \(\Lambda_{UV}\)

- **No free quarks** \(\rightarrow\) **Confinement**

\[Z(k^2 = M^2) = 0\]
**Off-Shell Effects**

- For an off-shell nucleon, photon–nucleon vertex given by

\[
\Gamma_{N}(p', p) = \sum_{\alpha, \beta=+, -} \Lambda_{\alpha}(p') \left[ \gamma_{\mu} f_{1}^{\alpha\beta} + \frac{i\sigma_{\mu\nu} q_{\nu}}{2M} f_{2}^{\alpha\beta} + q_{\mu} f_{3}^{\alpha\beta} \right] \Lambda_{\alpha}(p)
\]

- In-medium nucleon is off-shell, extremely difficult to quantify effects
  - However must understand to fully describe in-medium nucleon

- Simpler system: off-shell pion form factors
  - relax on-shell constraint \( p'^2 = p^2 = m_{\pi}^2 \)
  - Very difficult to calculate in many approaches, e.g. Lattice QCD

\[
(p' + p)^{\mu} F_{\pi,1}(p'^2, p^2, Q^2) + (p' - p)^{\mu} F_{\pi,2}(p'^2, p^2, Q^2)
\]

- For \( p'^2 = p^2 = m_{\pi}^2 \) we have \( F_{\pi,1} \to F_{\pi} \) and \( F_{\pi,2} = 0 \)
Results: Nuclear Matter

\[ \rho_p + \rho_n = \text{fixed} - \text{Differences arise from:} \]

- **naive:** different number protons and neutrons
- **medium:** \( p \) & \( n \) Fermi motion and \( V_{u(d)} \) differ \( \Rightarrow u_p(x) \neq d_n(x), \ldots \)