Higher Twist Scaling Violations

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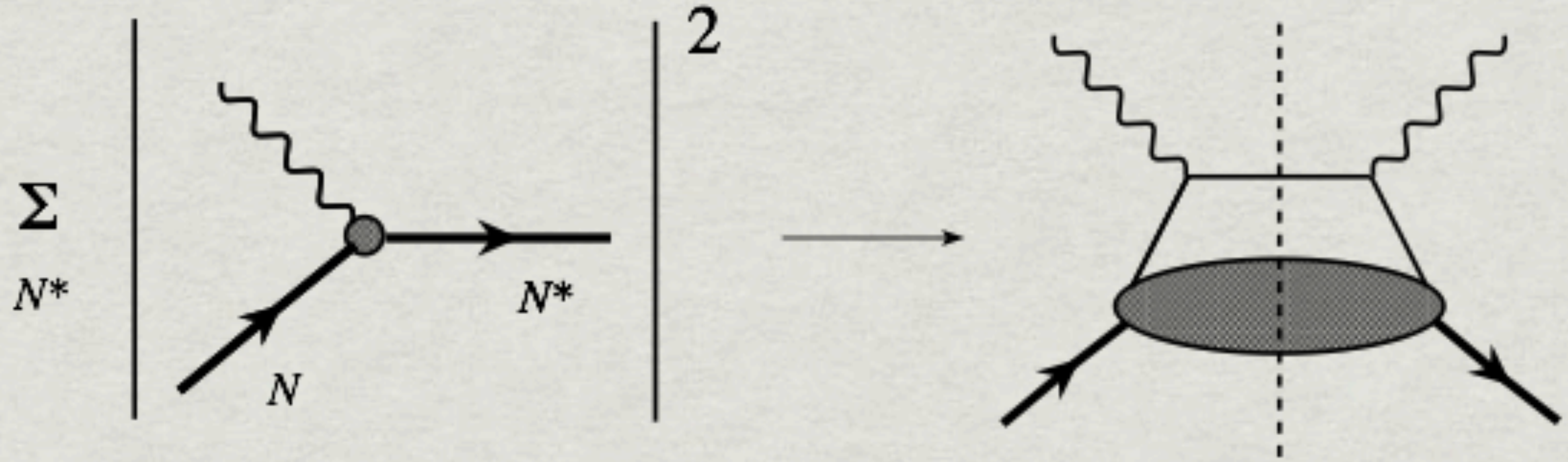
Outline

* Twist Expansion and Leading Twist
* Motivation
* Higher Twist
* Concluding Remarks

The Twist Expansion



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4}\frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$$

$$\boxed{\lim_{x\to 0} \mathrm{T}\{J_\mu(x) J_\nu(0)\} \sim \Gamma_{\mu\nu} \sum_{n,k} x^{\mu_1} \ldots x^{\mu_n} C_k^{(n)}(x^2) \mathcal{O}^{(n)}_{k,\mu_1\ldots\mu_n}(0)}$$

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The Cornwall-Norton Moment

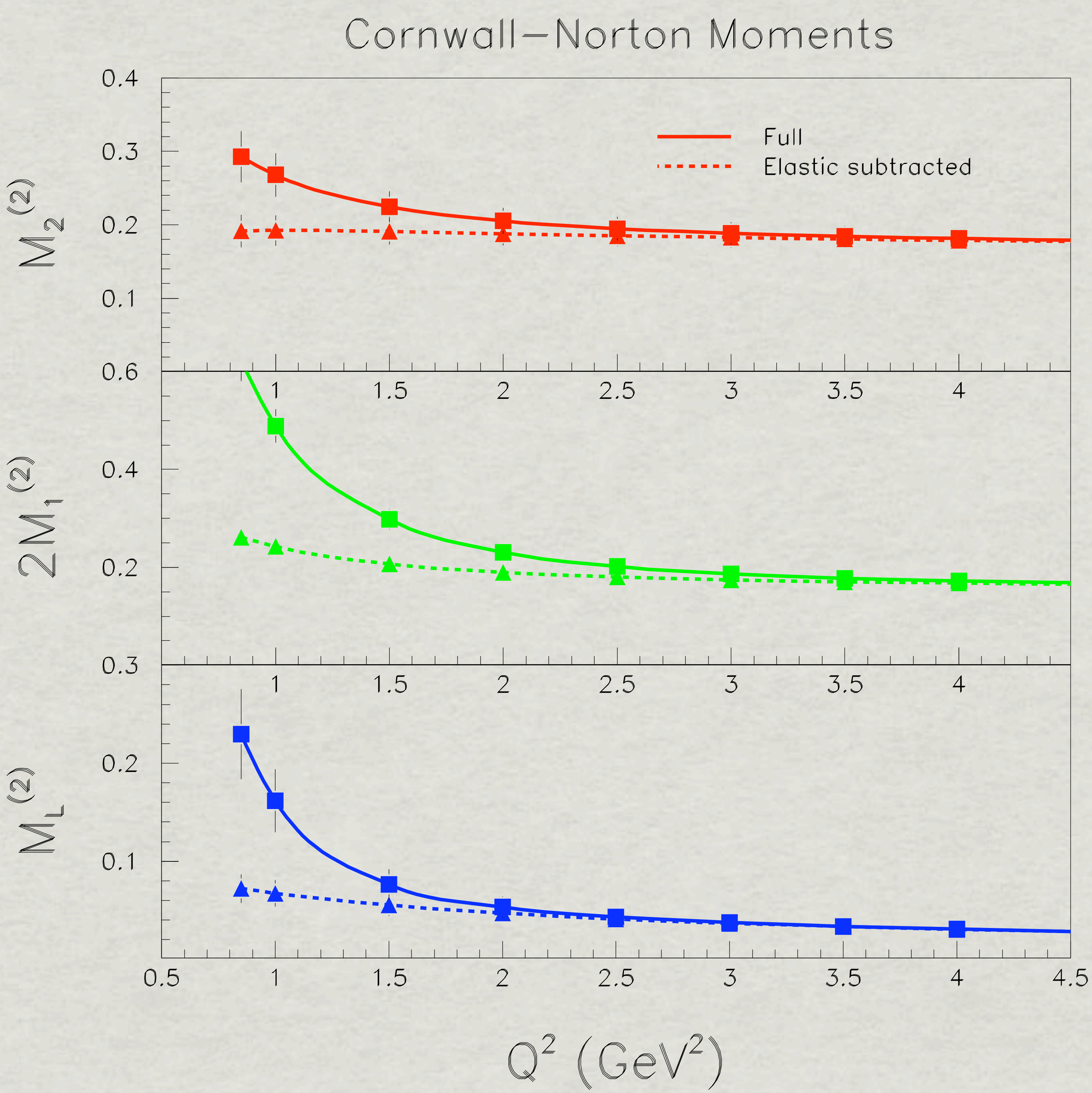
The moment of a structure function is a x-Bjorken weighted integral:

$$M^{(n)}(Q^2) = \int_0^1 dx_B \;\; x_B^{n-2} F_2(x_B, Q^2)$$

The moment can be computed using the momentum-space OPE:

$$\boxed{M_n(Q^2, g, \mu) = \int_0^1 dx_B \; x_B^{n-2} F_2(Q^2, g, \mu) \approx \sum_k \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \; \tilde{c}_k^n(g, \mu) \; A_k^{(n)}}$$

Leading Moment Data:

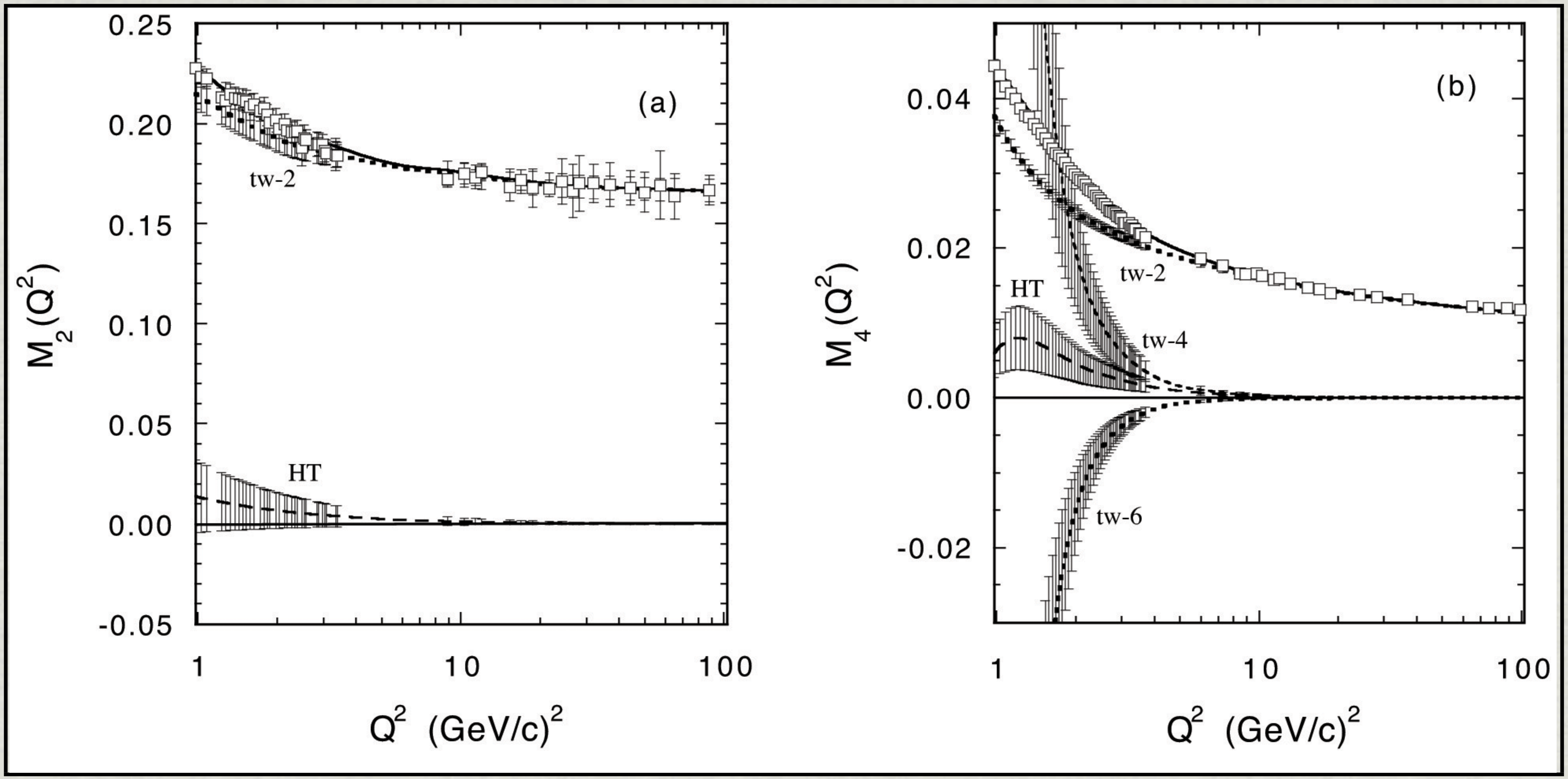


$$F_2^{\mathrm{EL}} = \frac{(G_E^2 + \tau G_M^2)\delta(x-1)}{1+\tau}$$

$$F_1^{\mathrm{EL}} = G_M^2 \delta(x-1) \qquad \tau = \frac{q^2}{4M_p^2}$$

$$F_L^{\mathrm{EL}} = G_E^2 \delta(x-1)$$

Cancellation of Higher Twist:



$$M_n(Q^2) = \eta_n(Q^2) + a_n^{(4)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(4)}} \frac{\mu^2}{Q^2} + a_n^{(6)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(6)}} \frac{\mu^4}{Q^4}$$

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Operator Basis for Twist-4

R.L. Jaffe & M. Soldate - Phys Rev D. V26 No. 1 (1982)

- We use a “canonical” basis of operators, proposed by Jaffe & Soldate:

- Twist-4 Operators must satisfy three conditions:
  a) Totally Symmetric
  b) Traceless
  c) Contain no contracted derivatives

- The first two conditions project out the highest spin of each operator.

- The last condition ensures the basis is not over-complete. These operators can be eliminated via QCD equations of motion.

The Operators:

- 14 Operators appear at twist-4, which can be divided into two groups: 6 four-quark operators, 8 two-quark operators, and pure-gluonic operators:

- Operators of the following form appear at twist-4:

$$\triangle \cdot Q_n^{1(k,l)} = g[\bar{\psi}_R \triangle \overleftarrow{d}^{\,l} \overrightarrow{d}^{\,k} \psi_R][\bar{\psi}_R \triangle \overrightarrow{d}^{\,n-2-k-l} \psi_R]$$

$$\triangle \cdot Q_n^{8(k)} = i\bar{\psi} \overleftarrow{d}^{\,k} f \overrightarrow{d}^{\,n-1-k} \psi$$

$$\triangle \cdot G_n^{(k,l)} = \mathrm{Tr}[f_\alpha \overrightarrow{d}^{\,n-k-l} f^\alpha \overrightarrow{d}^{\,k} f_\beta \overrightarrow{d}^{\,l} f^\beta]$$

- Notation:

$$\triangle \cdot \mathcal{O}_n \equiv \triangle^{\mu_1} \ldots \triangle^{\mu_n} \mathcal{O}_{n,\mu_1 \ldots \mu_n} \;; \qquad \triangle^2 = 0$$

$$d \equiv \triangle \cdot D$$

$$f^\beta \equiv F^{\rho\beta} \triangle_\rho$$

The Complete Basis:

$$\Delta\cdot Q_n^{1(k,l)} = g\bar{\psi}_R \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_R \bar{\psi}_R \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \psi_R \ ,$$

$$\Delta\cdot Q_n^{2(k,l)} = g\bar{\psi}_R \tau_a \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_R \bar{\psi}_R \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \tau_a \psi_R \ ,$$

$$\Delta\cdot Q_n^{3(k,l)} = g\bar{\psi}_R \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_R \bar{\psi}_L \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \psi_L \ ,$$

$$\Delta\cdot Q_n^{4(k,l)} = g\bar{\psi}_R \tau_a \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_R \bar{\psi}_L \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \tau_a \psi_L \ ,$$

$$\Delta\cdot Q_n^{5(k,l)} = g\bar{\psi}_L \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_L \bar{\psi}_L \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \psi_L \ ,$$

$$\Delta\cdot Q_n^{6(k,l)} = g\bar{\psi}_L \tau_a \not{\Delta} \overleftarrow{d}^{\,l} \overrightarrow{\mathrm{d}}^{k} \psi_L \bar{\psi}_L \not{\Delta} \overrightarrow{\mathrm{d}}^{n-2-k-l} \tau_a \psi_L \ ,$$

$$\Delta\cdot Q_n^{7(k)} = \bar{\psi} \overleftarrow{d}^{\,k} {}^{*}\!\not{f} \gamma_5 \overrightarrow{\mathrm{d}}^{n-1-k} \psi \ ,$$

$$\Delta\cdot Q_n^{8(k)} = i\bar{\psi} \overleftarrow{d}^{\,k} \not{f} \overrightarrow{\mathrm{d}}^{n-1-k} \psi \ ,$$

$$\Delta\cdot Q_n^{9(k,l)} = g\bar{\psi} \overleftarrow{d}^{\,k} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_a \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \psi \ ,$$

$$\Delta\cdot Q_n^{10(k,l)} = igf_{abc} \bar{\psi} \overleftarrow{d}^{\,k} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_b \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \tau_c \psi \ ,$$

$$\Delta\cdot Q_n^{11(k,l)} = gd_{abc} \bar{\psi} \overleftarrow{d}^{\,k} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_b \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \tau_c \psi \ ,$$

$$\Delta\cdot Q_n^{12(k,l)} = ig\bar{\psi} \overleftarrow{d}^{\,k} {}^{*} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_a \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \gamma_5 \psi \ ,$$

$$\Delta\cdot Q_n^{13(k,l)} = gf_{abc} \bar{\psi} \overleftarrow{d}^{\,k} {}^{*} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_b \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \gamma_5 \tau_c \psi \ ,$$

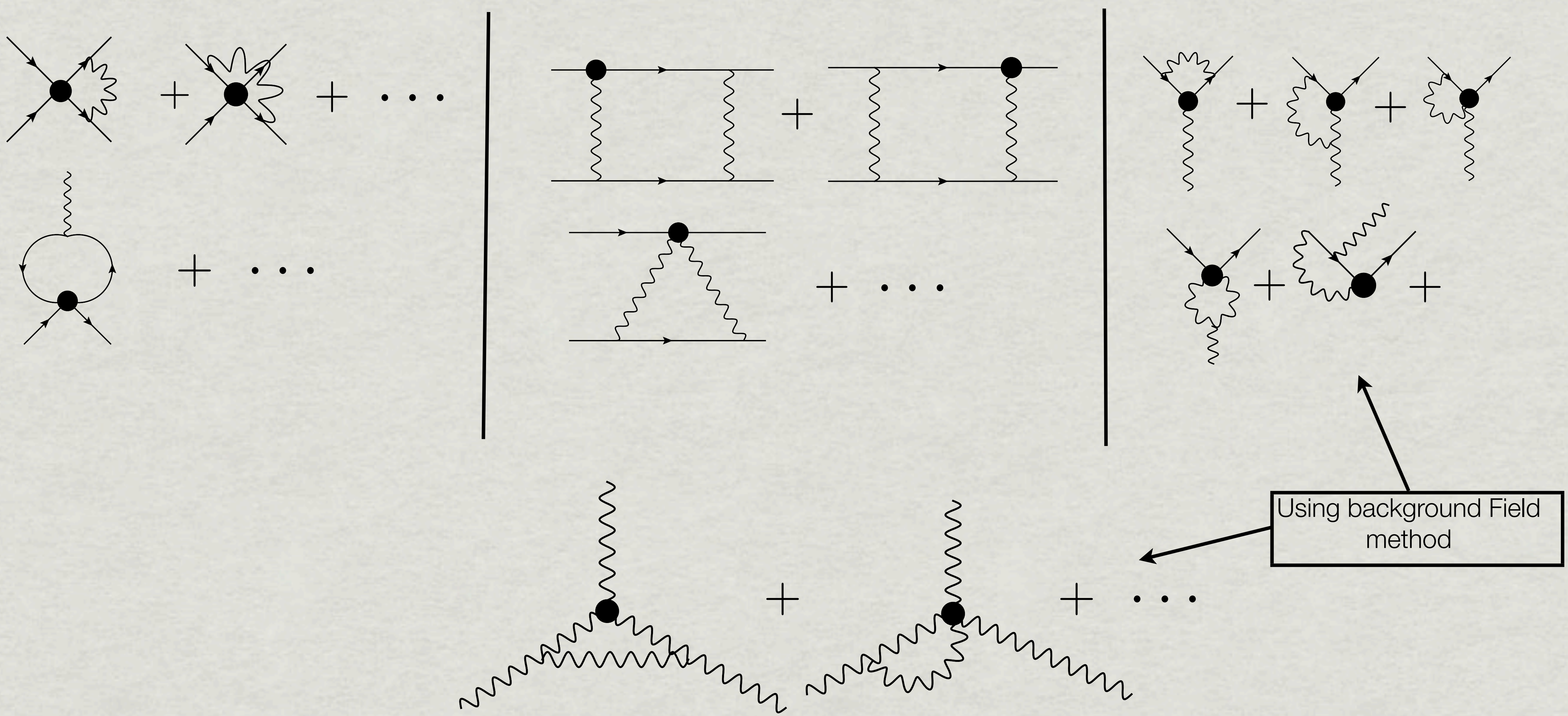
$$\Delta\cdot Q_n^{14(k,l)} = igd_{abc} \bar{\psi} \overleftarrow{d}^{\,k} {}^{*} f_a^{\alpha} (\overrightarrow{\mathrm{d}}^{l} f_{\alpha})_b \overrightarrow{\mathrm{d}}^{n-3-k-l} \not{\Delta} \gamma_5 \tau_c \psi \ .$$

$$\triangle \cdot O_n^{G1(k,l)} = \mathrm{Tr}[f_\alpha \overrightarrow{d}^{\,n-2-k-l} f^\alpha \overrightarrow{d}^{\,k} f_\beta \overrightarrow{d}^{\,l} f^\beta]$$

$$\triangle \cdot O_n^{G2(k,l)} = \mathrm{Tr}[F^{\alpha\beta} \overrightarrow{d}^{\,n} F_{\alpha\beta}]$$

The Anomalous Dimension:

- In order to RG evolve the moments of the structure functions, we must compute the one-loop corrections to each operator



Preliminary Results:

$$Z_{ij} \sim \begin{pmatrix} \text{Quark} & \text{Quark} \to \text{Glue} \\ \text{Glue} \to \text{Quark} & \text{Glue} \end{pmatrix}$$

$$Z_{\text{Quark}} \sim \begin{pmatrix}
-\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{g^2}{36\pi^2\epsilon} & -\frac{7g^2}{72\pi^2\epsilon} & 0 & \frac{g^2}{18\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{g^2}{48\pi^2\epsilon} & \frac{g^2}{36\pi^2\epsilon} & \frac{g^2}{48\pi^2\epsilon} & 0 & -\frac{g^2}{36\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{g^2}{18\pi^2\epsilon} & -\frac{g^2}{36\pi^2\epsilon} & -\frac{g^2}{72\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
\frac{65g^2}{96\pi^2\epsilon} & \frac{19g^2}{48\pi^2\epsilon} & \frac{485g^2}{432\pi^2\epsilon} & \frac{31g^2}{36\pi^2\epsilon} & \frac{163g^2}{288\pi^2\epsilon} & \frac{29g^2}{48\pi^2\epsilon} & \frac{239g^2}{576\pi^2\epsilon} & \frac{83g^2}{288\pi^2\epsilon} & -\frac{11g^2}{144\pi^2\epsilon} & \frac{11g^2}{144\pi^2\epsilon} \\
\frac{25g^2}{144\pi^2\epsilon} & \frac{47g^2}{576\pi^2\epsilon} & \frac{25g^2}{216\pi^2\epsilon} & \frac{67g^2}{288\pi^2\epsilon} & \frac{g^2}{16\pi^2\epsilon} & \frac{167g^2}{576\pi^2\epsilon} & -\frac{19g^2}{96\pi^2\epsilon} & -\frac{671g^2}{1152\pi^2\epsilon} & \frac{5g^2}{144\pi^2\epsilon} & -\frac{97g^2}{576\pi^2\epsilon} \\
-\frac{181g^2}{864\pi^2\epsilon} & \frac{59g^2}{576\pi^2\epsilon} & -\frac{181g^2}{432\pi^2\epsilon} & \frac{77g^2}{288\pi^2\epsilon} & -\frac{181g^2}{864\pi^2\epsilon} & \frac{59g^2}{576\pi^2\epsilon} & -\frac{11g^2}{192\pi^2\epsilon} & -\frac{g^2}{576\pi^2\epsilon} & \frac{175g^2}{288\pi^2\epsilon} & -\frac{g^2}{288\pi^2\epsilon} \\
\frac{97g^2}{432\pi^2\epsilon} & -\frac{55g^2}{576\pi^2\epsilon} & \frac{101g^2}{216\pi^2\epsilon} & -\frac{5g^2}{32\pi^2\epsilon} & \frac{97g^2}{432\pi^2\epsilon} & -\frac{55g^2}{576\pi^2\epsilon} & -\frac{g^2}{64\pi^2\epsilon} & -\frac{41g^2}{576\pi^2\epsilon} & \frac{35g^2}{144\pi^2\epsilon} & \frac{175g^2}{288\pi^2\epsilon}
\end{pmatrix}
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7^{k=0} \\ Q_7^{k=1} \\ Q_8^{k=0} \\ Q_8^{k=1} \end{pmatrix}$$

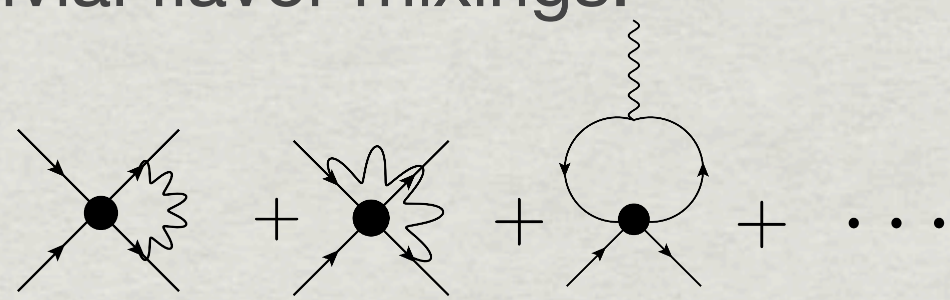
$$Z_{\text{Glue}} \sim \begin{pmatrix} \frac{g^2}{3\pi^2\epsilon} & -\frac{g^2}{\pi^2\epsilon} \\ 0 & \frac{31g^2}{64\pi^2\epsilon} \end{pmatrix} \begin{pmatrix} \mathcal{G}_2^{1(1,1)} \\ \mathcal{G}_2^{2(1,1)} \end{pmatrix}$$

PV-DIS Result:

- The relevant twist-four operator for PVDIS takes the following form:

$$\mathcal{O}^{\mu\nu}_{\mathrm{ud}} \sim \bar{u}(x)\gamma^{\mu}u(x)\bar{d}(0)\gamma^{\nu}d(0)$$

- The 1-loop corrections to this operator (from 4-quark graphs) generate non-trivial flavor mixings:



$$\boxed{\mathcal{O}^{\mu\nu}_{\mathrm{ud}} \rightarrow \frac{g^2(18\mathcal{O}^{\mu\nu}_{\mathrm{uu}} + 18\mathcal{O}^{\mu\nu}_{\mathrm{dd}} - 133\mathcal{O}^{\mu\nu}_{\mathrm{ud}} + 18\mathcal{O}^{\mu\nu}_{\mathrm{us}} + 18\mathcal{O}^{\mu\nu}_{\mathrm{ds}})}{144\pi^2\epsilon}}$$

Concluding Remarks:

- DGLAP evolution equations can be incorporated by leading twist, but there is no DGLAP analog for higher twist, we expect power corrections to scaling at lower Q^2.
- At twist-4, these scaling violations include involved mixings among many operators.
- We hope to extend this analysis to arbitrary spin, employing a technique called non-local operator renormalization.