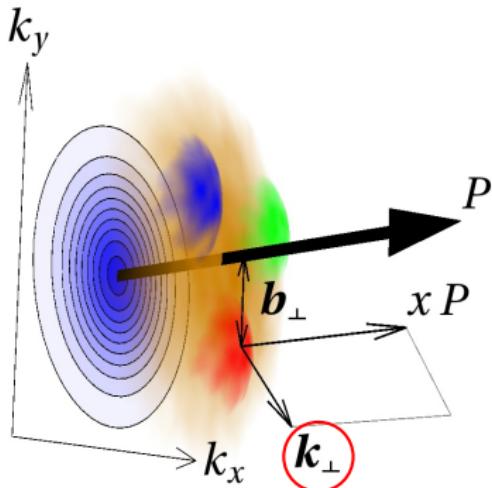


# **Prospects of TMD PDF predictions from Lattice QCD**

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in collaboration with

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and the LHP Collaboration



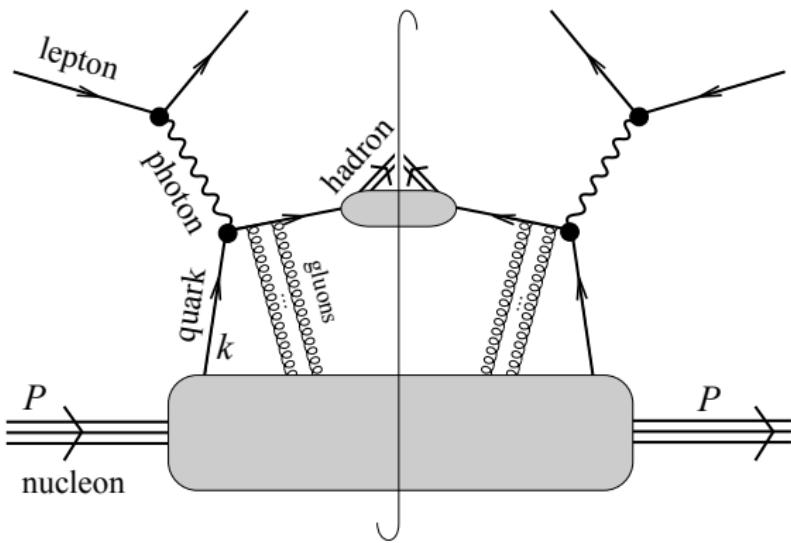
## TMD PDFs

transverse momentum dependent  
parton distribution functions

e.g.,  $f_1(x, \mathbf{k}_\perp^2)$

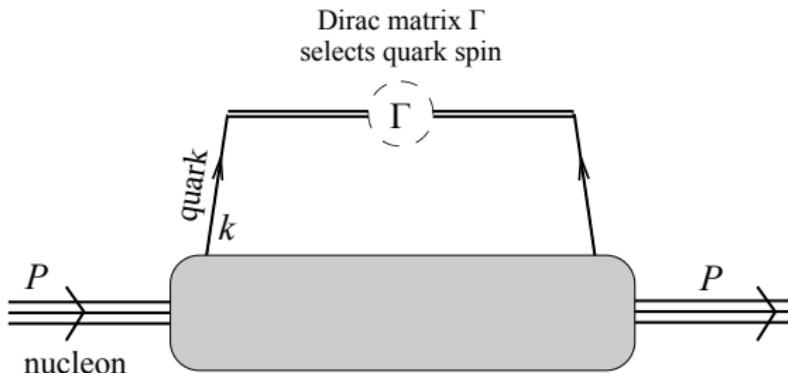
⇒ quark density  $\rho(\mathbf{k}_\perp)$ .

- $x$ , ⇒ PDFs
- $\mathbf{b}_\perp$  (impact parameter), ⇒ GPDs
- $\mathbf{k}_\perp$  (intrinsic quark transverse momentum) ⇒ TMD PDFs



$$\frac{d\sigma}{d^3 P_h d^3 P_{l'}} \propto \underbrace{L_{\mu\nu}}_{\substack{\text{lepton} \\ \text{tensor}}} \quad \underbrace{W^{\mu\nu}}_{\substack{\text{hadron} \\ \text{tensor}}}$$

$$W_{\text{unpol., LO}}^{\mu\nu} \propto \int d\ell_\perp e^{i\ell_\perp \cdot \mathbf{P}_{h\perp}} \underbrace{\tilde{f}_1(x, z\ell_\perp, \dots)}_{\substack{\text{TMD PDF}}} \underbrace{\tilde{D}_h(z, \ell_\perp, \dots)}_{\substack{\text{fragmentation f.}}} \underbrace{\tilde{H}(Q^2, \dots)}_{\substack{\text{hard part}}}$$

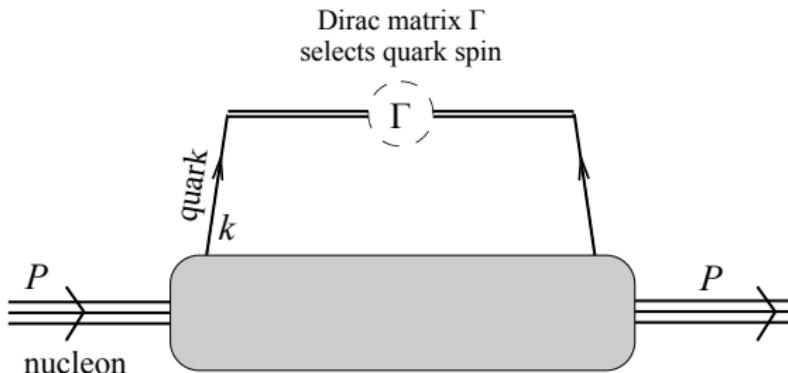


$$\Phi^{[\Gamma]}(k, P, S) \equiv “\langle P, S | \bar{q}(k) \Gamma q(k) | P, S \rangle”$$

lightcone coor.  $w^\pm = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$ , so  $w = w^+ \hat{n}_+ + w^- \hat{n}_- + w_\perp$   
 proton flies along z-axis:  $P^+$  large,  $P_\perp = 0$

parametrization in terms of TMD PDFs, example

$$\int dk^- \Phi^{[\gamma^+]}(k, P, S) \Big|_{k^+ = xP^+} = f_1(x, \mathbf{k}_\perp^2) + \langle \text{spin dep. terms} \rangle$$



$$\Phi^{[\Gamma]}(k, P, S) \equiv \frac{1}{2} \int \frac{d^4\ell}{(2\pi)^4} e^{-ik\cdot\ell} \langle P, S | \bar{q}(\ell) \Gamma \mathcal{U} q(0) | P, S \rangle$$

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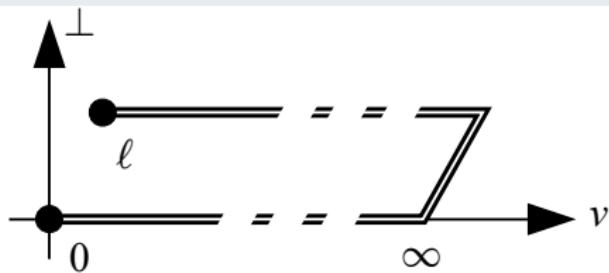
parametrization in terms of TMD PDFs, example

$$\int dk^- \Phi^{[\gamma^+]}(k, P, S) \Big|_{k^+ = xP^+} = f_1(x, \mathbf{k}_\perp^2) + \langle \text{spin dep. terms} \rangle$$

$$\mathcal{U} \equiv \mathcal{P} \exp \left( -ig \int_0^\ell d\xi^\mu A_\mu(\xi) \right) \quad \text{along path from } 0 \text{ to } \ell$$

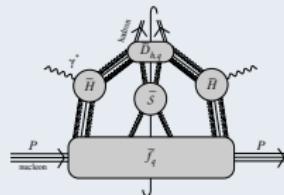
$\implies \langle P | \bar{q}(\ell) \Gamma \mathcal{U} q(0) | P \rangle$  is gauge invariant.

SIDIS / Drell Yan



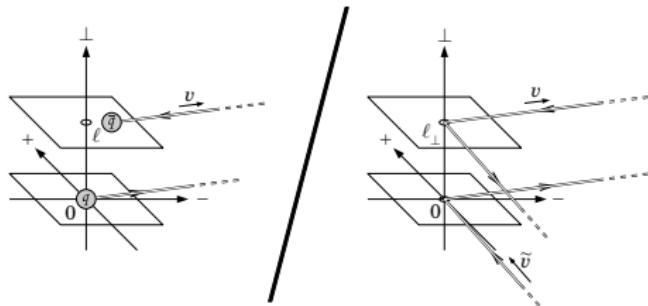
$v = \hat{n}_-$  (lightlike)

$$W_{\text{unpol., LO}}^{\mu\nu} \propto \int d\ell_\perp e^{i\ell_\perp \cdot P_{h\perp}} \\ \times \tilde{f}_1(\dots) \tilde{D}_h(\dots) \tilde{H}(\dots) \underbrace{\tilde{S}(\ell_\perp, \dots)}_{\text{soft factor}}$$



modified definition of TMD PDF correlator:

$$\Phi^{[\Gamma]}(k, P, S) \equiv \frac{1}{2} \int \frac{d^4 \ell}{(2\pi)^4} e^{-ik \cdot \ell} \frac{\langle P, S | \bar{q}(\ell) \Gamma U q(0) | P, S \rangle}{\tilde{S}(\ell_\perp, \dots)}$$



- gauge links slightly off lightcone:  $v \neq \hat{n}_-$
- ⇒ evolution eqn. in  $\zeta \equiv (v \cdot P)^2/v^2$
- soft factor  $\tilde{S}$ : vacuum expectation value of gauge link structure

## solved

- factorization shown.
- “rapidity divergences” removed by use of non-lightlike gauge link.

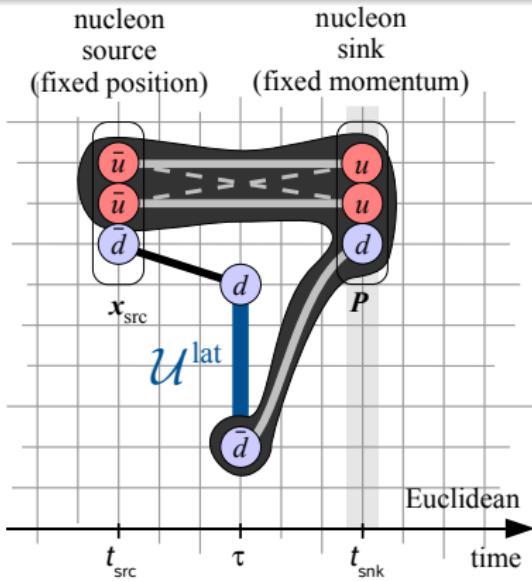
## open issues

- UV divergence from gauge link self energy:

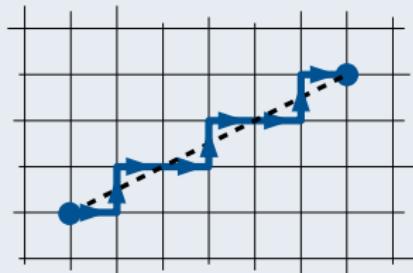
$$\mathcal{U}^{\text{ren}} = \mathcal{U} \exp(\delta m \langle \text{link length} \rangle)$$

## wish list

- $f_1(x) = \int d^2\mathbf{k}_\perp f_1(x, \mathbf{k}_\perp^2)$
- probability interpretation



## gauge link on lattice



For now, approximate  
**straight** gauge link.  
⇒ no  $T$ -odd structures  
(Sivers, Boer-Mulders fcn.)

extract Lorentz-invariant amplitudes  $\tilde{A}_i(\ell^2, \ell \cdot P)$

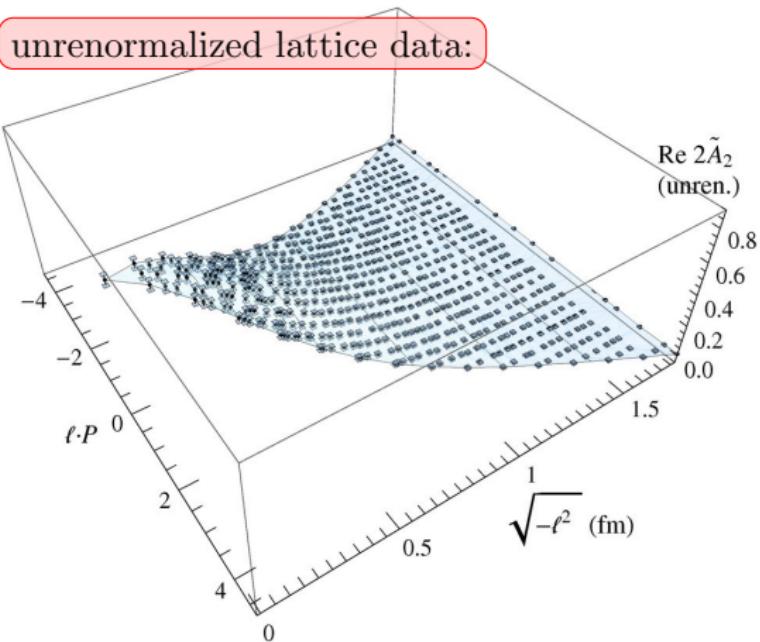
$$\langle P, S | \bar{q}(\ell) \gamma_\mu \mathcal{U} q(0) | P, S \rangle = 4 \tilde{A}_2 P_\mu + 4i m_N^2 \tilde{A}_3 \ell_\mu$$

$\Rightarrow f_1(x, \mathbf{k}_\perp^2)$

Amplitudes are complex and fulfill  $[\tilde{A}_i(\ell^2, \ell \cdot P)]^* = \tilde{A}_i(\ell^2, -\ell \cdot P)$ .  
Operator must not have temporal extent:  $\ell^0 = \ell_4 = 0$ .

$$f_1(x, \mathbf{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x, \mathbf{k}_\perp; P, S)$$

$$= \int \frac{d(\ell \cdot P)}{2\pi} e^{ix(\ell \cdot P)} \int \frac{d^2 \ell_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \ell_\perp} 2\tilde{A}_2(\ell^2, \ell \cdot P) \Big|_{\ell^+ = 0}$$

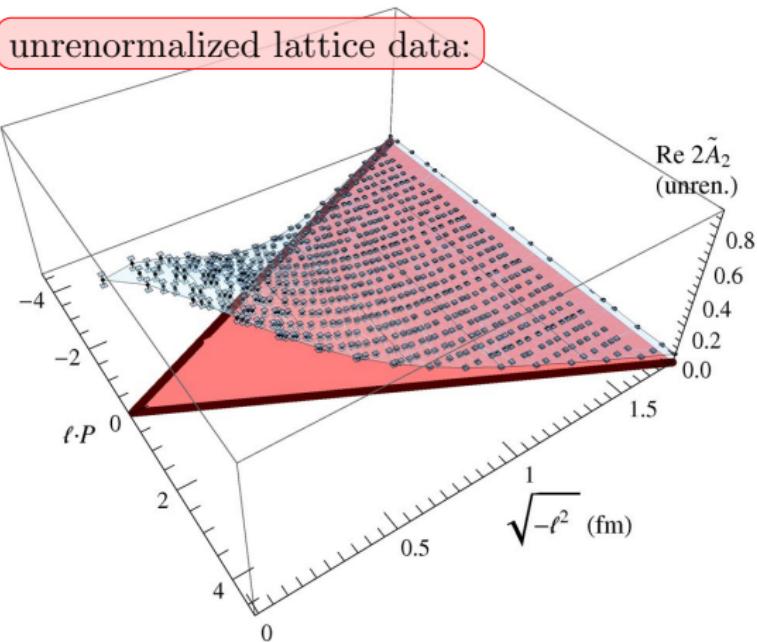


$$\ell^2 \xleftrightarrow{\text{FT}} \mathbf{k}_\perp^2$$

$$\ell \cdot P \xleftrightarrow{\text{FT}} x$$

$$f_1(x, \mathbf{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x, \mathbf{k}_\perp; P, S)$$

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$$\ell^2 \xleftrightarrow{\text{FT}} \mathbf{k}_\perp^2$$

$$\ell \cdot P \xleftrightarrow{\text{FT}} x$$

Euclidean lattice

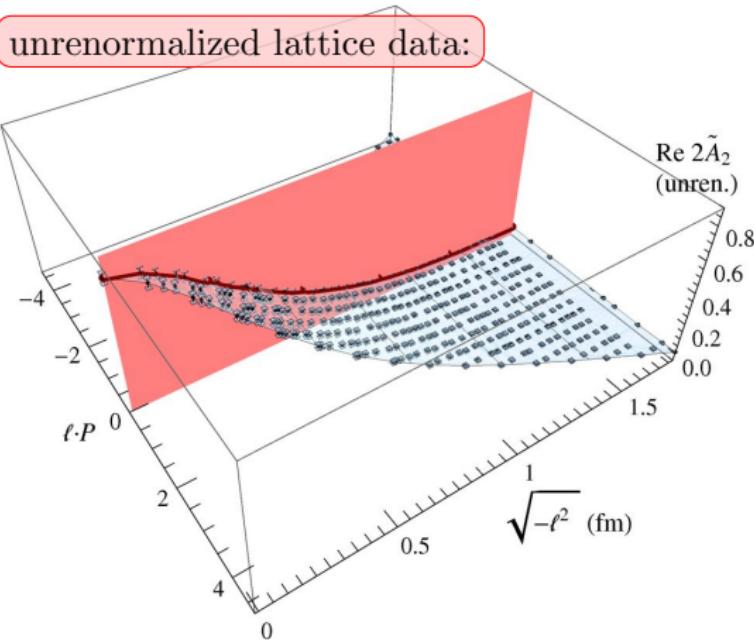
$$\ell_4 = 0$$

$$\Downarrow$$

$$\begin{aligned} \ell^2 &\leq 0, \\ |\ell \cdot P| &\leq |\mathbf{P}| \sqrt{-\ell^2} \end{aligned}$$

$$f_1(x, \mathbf{k}_\perp^2) \equiv \Phi^{[\gamma^+]}(x, \mathbf{k}_\perp; P, S)$$

$$= \int \frac{d(\ell \cdot P)}{2\pi} e^{ix(\ell \cdot P)} \int \frac{d^2 \ell_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \ell_\perp} 2\tilde{A}_2(\ell^2, \ell \cdot P) \Big|_{\ell^+ = 0}$$



$$\begin{aligned} \ell^2 &\xleftrightarrow{\text{FT}} \mathbf{k}_\perp^2 \\ \ell \cdot P &\xleftrightarrow{\text{FT}} x \end{aligned}$$

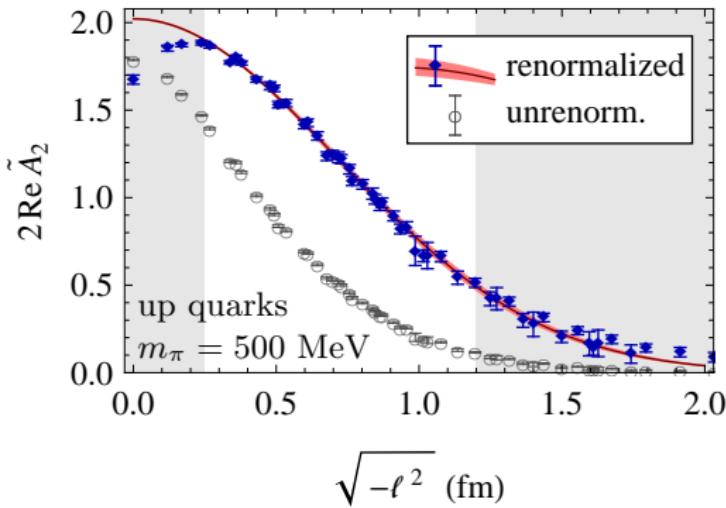
Euclidean lattice

$$\ell_4 = 0$$

$$\Downarrow$$

$$\begin{aligned} \ell^2 &\leq 0, \\ |\ell \cdot P| &\leq |\mathbf{P}| \sqrt{-\ell^2} \end{aligned}$$

$$\text{lowest } x\text{-moment} \int_{-1}^1 dx f_1(x, \mathbf{k}_\perp^2) \equiv \int dx \int dk^- \Phi^{[\gamma^+]}(k, P, S) \\ = \int \frac{d^2 \ell_\perp}{(2\pi)^2} e^{i \mathbf{k}_\perp \cdot \ell_\perp} 2 \tilde{A}_2(-\ell_\perp^2, 0)$$



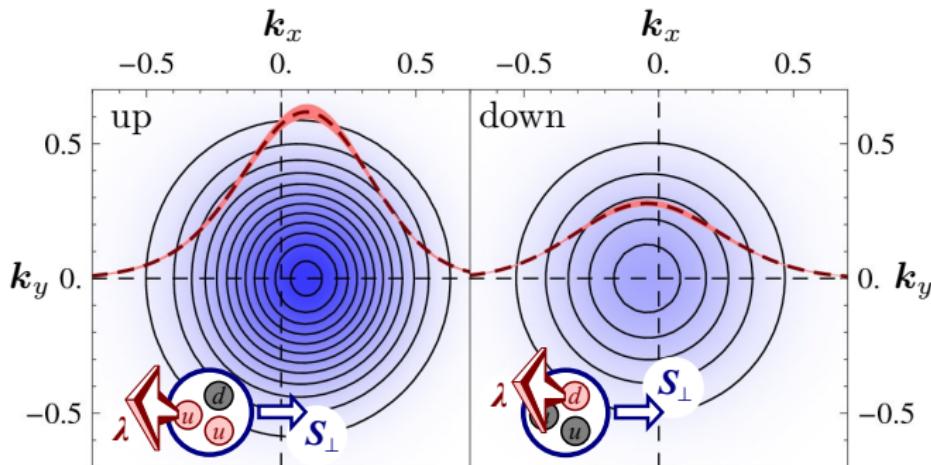
fit function

$$C_1 \exp(-|\ell|^2/\sigma_1^2)$$

Exclude data below  
 $\sqrt{-\ell^2} \leq 0.25$  fm:  
lattice cutoff effects.  
Expect continuum  
divergence at  $\ell = 0$ .  
Gaussian  $\hat{=}$  regulator.

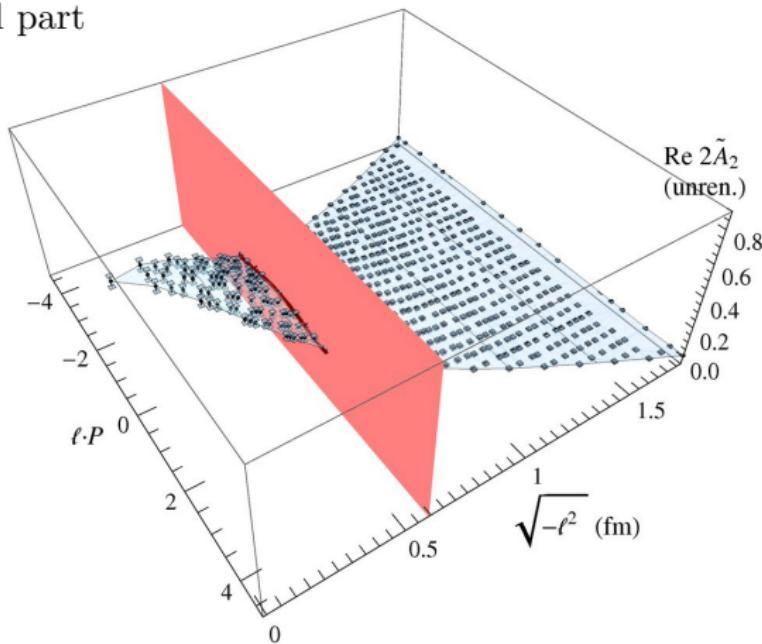
Density of quarks with positive helicity,  $\lambda = 1$ ,  
in a transversely polarized nucleon,  $\mathbf{S}_\perp = (1, 0)$ :

$$\begin{aligned}\rho_{TL}(\mathbf{k}_\perp; \mathbf{S}_\perp, \lambda) &\equiv \frac{1}{2} \int dx \int dk^- \Phi^{[\gamma^+ \frac{1}{2}(1+\gamma^5)]}(k, P, S_\perp) \\ &= \frac{1}{2} f_1^{(1)sW}(\mathbf{k}_\perp^2) + \frac{\lambda}{2} \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m_N} g_{1T}^{(1)sW}(\mathbf{k}_\perp^2)\end{aligned}$$



$(m_\pi \approx 500 \text{ MeV}, \text{ straight gauge link operator, } C^{\text{ren}} = 0, \text{ Gaussian fit})$

real part

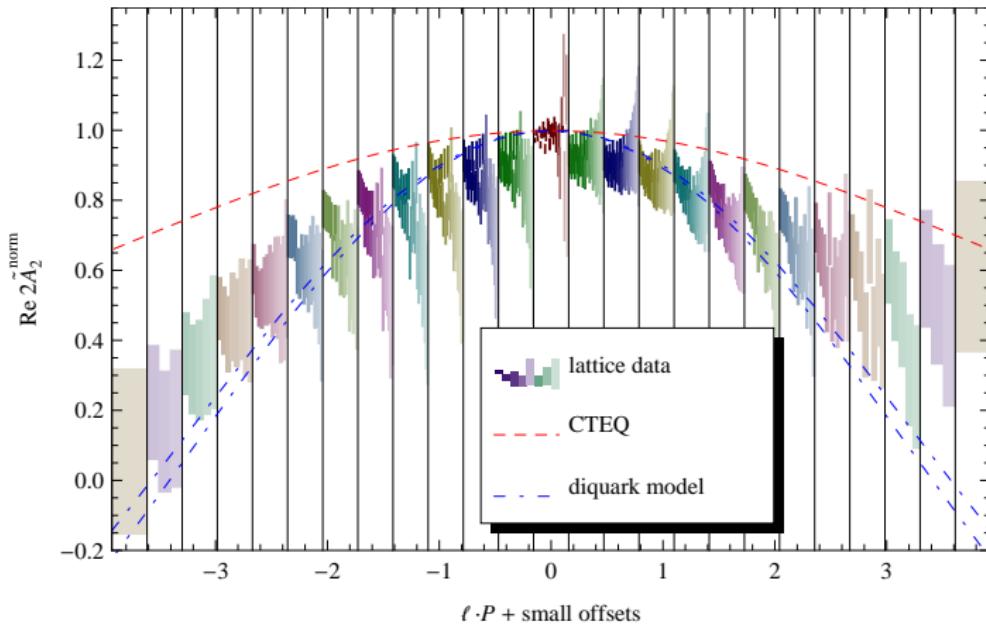


$$\ell^2 \xleftrightarrow{\text{FT}} k_{\perp}^2$$

$$\ell \cdot P \xleftrightarrow{\text{FT}} x$$

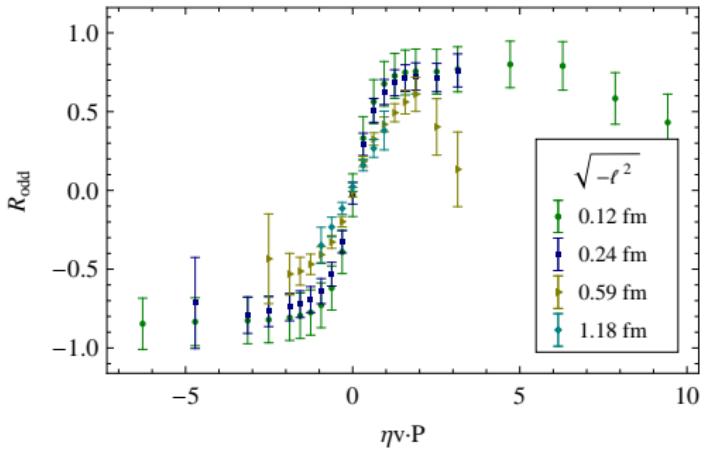
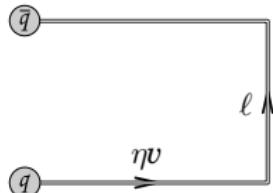
Within our limited  $(\ell^2, \ell \cdot P)$  window,  
our lattice data are compatible with the assumption

$$f_1(x, \mathbf{k}_\perp^2) = f_1(x) \hat{f}_1(\mathbf{k}_\perp^2)$$



**ratio** of amplitudes with staple like gauge links

$$R_{\text{odd}} \xrightarrow{\eta \text{ large}} \frac{\left[ \tilde{A}_{12} + \frac{m_N^2}{P_1^2} \tilde{B}_8 \right] (\ell^2, 0, 0, \zeta^{-1}, \pm 1)}{\tilde{A}_2(\ell^2, 0, 0, \zeta^{-1}, \pm 1)}$$



Part of the effect comes from the Sivers function  $f_{1T}^\perp$  via  $\tilde{A}_{12}$ !

$$\zeta = (v \cdot P)^2 / v^2$$

$\zeta \rightarrow \infty$  for lightlike links.

Need to evolve lattice results to high  $\zeta$ .

Presently  
 $\zeta^{\text{lat}} \approx (0.5 \text{ GeV})^2$ .

$$\zeta_{\text{max}}^{\text{lat}} \propto (P^{\text{lat}})^2$$

Need high nucleon momenta on lattice.  
Difficult (noisy).

## present: straight gauge links

- not directly comparable to experimental situation
- ⇒ qualitative statements
- comparison to models, test of phenomenological assumptions
- easy access to spin-dependence

## near future: staple shaped links, ratios of amplitudes

- question:  $\zeta$  large enough on lattice?
- size estimates of spin-dependent and T-odd effects (Sivers,...)

## prerequisite for quantitative lattice predictions

“To allow non-perturbative methods in QCD to be used to estimate parton densities, operator definitions of parton densities are needed that **can be taken literally.**” [COLLINS arXiv:0808.2665 (2008)]

## known fundamental limitations of (our) lattice TMD PDF calculations

- Euclidean space ⇒ limited  $(\ell \cdot P)$ -range ⇒ limited access to  $x$ -depend.
- evolution parameter  $\zeta \propto (\text{lattice nucleon momentum})^2$